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The reflected sound field by curved surfaces

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Many spaces have curved walls or ceilings. With improved building technology and new fashions in architecture (blobs) there is an increasing number of problems due to the acoustic reflections by these surfaces. Sound reflected by concave surfaces will concentrate in a narrow area.
In practical applications of room acoustics these curved surfaces will be calculated with mirror imaging or ray tracing programs, in which the structure is modeled by flat segments. Alternative is a geometrical approach. Both methods do not correspond to reality.
The only valid calculation method is the calculation from a wave extrapolation method. It is shown that a theoretical correct solution of the sound field by curved surfaces is possible. A fairly simple expression for the sound pressure in the focal point is found and a more complicated description of the reflected sound field by small curved surfaces is presented. With these results the sound field in field applications can be calculated.

Wigmore Hall London	Royal Albert Hall London	Tonhalle Düsseldorf(before renovation)

Figure 1. Some examples of concert halls with curved surfaces.

1 Introduction

Many small or large rooms have concave surfaces. With improved building technology and fashions in architecture (blobs) problems due to these surfaces are encountered more and more. Some situations are described in literature [1,2,3]. In our consultancy work we had to deal with these situations e.g. in concert halls (figure 1 and [4,5]).

When sound is reflected from a concave surfaces the geometry of the surface will force the energy to concentrate. Figure 2 shows the impuls response (energy-time-curve ETC) of the Tonhalle Düsseldorf , before renovation. We see that a very significant echo occurs.

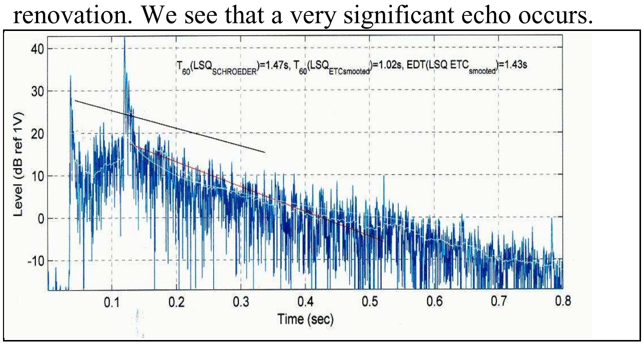


Figure 2. example of the impulse response in Tonhalle Düsseldorf (before renovation).

The sound pressure due to this focussing if mostly calculated by computer simulation techniques applied on a segmented shape or by a geometrical approximation. Both methods however fail in the focussing point, the result is not even close to the real value. In the geometrical approach the pressure will go to infinity.

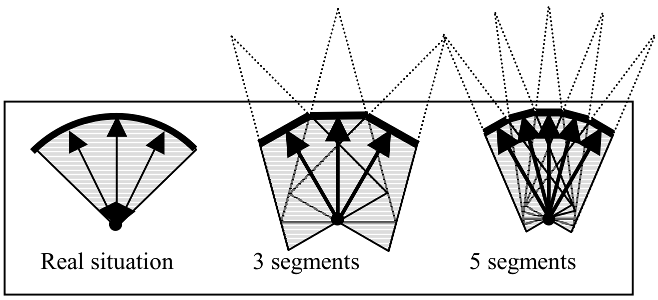


Fig.3 Illustration of the different distribution of pressure when using image sources, depending on the segmentation of the curved surface.

When using segmented shapes and image sources the calculated pressure in the focal point depends on the segmentation in the model. (see figure 3).

This rises the (first) question what sound pressure to expect in the focal point. In a number of consultancy projects, such as the two shown in figure 2, it was found very difficult to sufficiently reduce the echo's found. So this rises the second question we will try to answer in this paper, why it is that these echo's are so persistent and to what level the can be reduced. Not only the pressure in the focal point is important. As a third item we will derive a more general approximation of the sound field from curved surfaces.

To answer the first question, to be able to correctly estimate the sound pressure in the focal point, a wave extrapolation method is used, that will be presented here.

2 Wave extrapolation

Wave extrapolation uses the Huygens principle, developed by Christiaan Huygens in 1678 and later improved by Fresnal. The Huygens Principle states that every point on the primary wavefront can be thought of as an emitter of secondary wavelets. The secondary wavelets combine to produce a new wavefront in the direction of propagation. Fresnel extended the theory of Huygens in stating that the secondary wavelets mutually interfere. But it was Kirchhoff who put the Huygens-Fresnel principle on a sounder mathematical basis.
From Green's theorem the Kirchhoff integral can be derived:

$$P_A = \frac{1}{4\pi}\int_S \left(P(r)\frac{1+jkD}{D}\cos\varphi\frac{e^{-jkD}}{D} + j\omega\omega\cdot v_n(r)\frac{e^{-jkD}}{D}\right)dS =$$

$$\frac{\hat{p}}{4\pi}\int_S \left(\frac{1+jkD}{D}\cos\varphi + \frac{1+jkr}{r}\cos\alpha\right)\frac{e^{-jk(D+r)}}{Dr}dS \qquad (1)$$

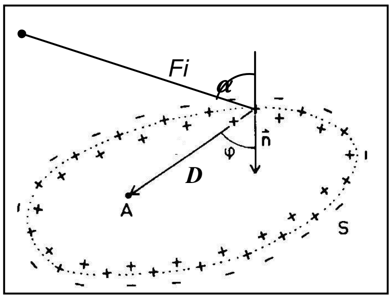


Fig.4 Symbols used for the Kirchhoff integral

It states that, with a sound source outside volume V, the sound pressure at point *A* inside *V* can be calculated from the sound pressure and particle velocity data on the surface *S* that is the boundary of *V*.
In the integral above the sound pressure and particle velocity is assumed to be known and resulting from a source at distance r from the surface element *dS*.

The distance between *dS* and *A* is given by D. The angles α and φ are the angles between the incident sound and the (outer)normal to *dS* and between the (inner)normal to *dS* and the vector to *A* from *dS* respectively.

For $r >> \lambda$ and $d >> \lambda$ (far field) this can be simplified:

$$P_A = -\frac{j\hat{p}}{\lambda}\int_S \left(\frac{\cos\alpha + \cos\varphi}{2}\right)\frac{e^{-jk(D+r)}}{Dr}dS \qquad (2)$$

This is known as the Fresnel-Kirchhoff diffraction formula. In case of wave propagation from a flat surface the Kirchhoff integral may be further simplified to the Rayleigh integral, either using the sound pressure data or the particle velocity data. Since we will calculate the reflection by extrapolating the wavelets on concave surfaces we will use the Kirchhoff integral. This is done by first calculating the wavefield from the source on the concave surface and then (without the source and with the velocity at opposite phase) integrating the contributions of the wavelets over the concave surface, thus obtaining (only) the reflected sound.

3 Reflection in the focal point from a sphere

We take a full sphere with radius R and describe it with the spherical coordinates (r,ϕ,θ) and the surface elements $dS = R^2 \cdot \sin\phi \cdot d\phi \cdot d\theta$.

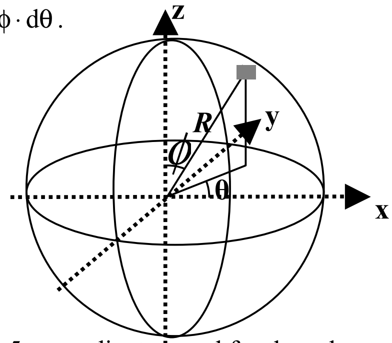


Figure 5. co-ordinates used for the sphere

If the sound source is in the origin, the sound pressure at the sphere's surface will be:

$$P(R,\omega) = \hat{p}\frac{e^{-jkR}}{R} \qquad (3)$$

Integrating over the sphere will give the following (Kirchhoff) integral expression for the pressure in a point *A* inside the sphere:

$$P_A = \frac{\hat{p}R\cdot e^{-jkR}}{4\pi}\int_0^{\pi}\int_0^{2\pi}\left(\frac{1+jkR}{R}\cos\alpha + \frac{1+jkD}{D}\cos\varphi\right)\frac{e^{-jkD}}{D}\sin\phi\cdot d\phi d\theta \qquad (4)$$

For the source in the origin $\cos\alpha = 1$. For point $A = (0,0,0)$ an analytical solution is possible:

$$P_A = -\frac{j\hat{p}R}{\lambda}\cdot e^{-jkR}\int_0^{\pi}\int_0^{2\pi}\sin\phi\cdot\frac{e^{-jkR}}{R}\cdot d\phi d\theta = -j2\hat{p}k\cdot e^{-j2kR}$$

The amplitude of the reflecting wave in the centre of the sphere is: $|P_A| = 2k\hat{p} = \frac{4\pi\hat{p}}{\lambda}$ (5)

This amplitude corresponds roughly with the amplitude one would get if the reflected energy is distributed over an circular area with radius $\frac{1}{4}\lambda$.

Since all contributions of *dS* are in phase, this amplitude is proportional to the reflecting surface that contributes. For a hemisphere the amplitude in the center will be:

$$|P_A| = k\hat{p}\text{, or } p_{eff}^2 = \tfrac{1}{2}\hat{p}^2k^2 \qquad (6)$$

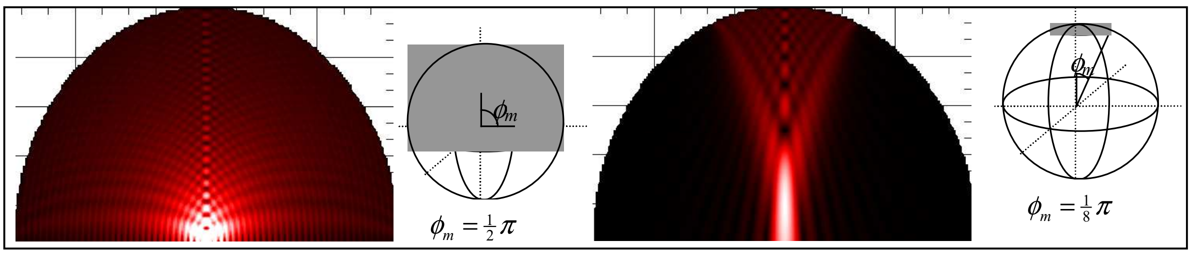


Figure 6. reflected sound pressure from a dome segment; R=10m, k=10 (f=500Hz); black=0, white ≥ 1.

For a sphere fragment integrating ϕ from 0 to ϕ_m this will result in:

$$|P_A| = k\hat{p}(1-\cos\phi_m) = \frac{2\pi\hat{p}(1-\cos\phi_m)}{\lambda} \qquad (7)$$

In figure 6 two examples of the calculated pressure is given for different ϕ_m.

4 Reflection in the focal line from a cylinder

The reflected sound field from a cylinder can, as for the dome, be described by the Kirchhoff integral.

The positions within the cylinder are described by cylindrical co-ordinates (r,θ,z) and the surface elements on the cylinder have dimension $dS = Rd\theta dz$. The source is assumed in the center (0,0,0).

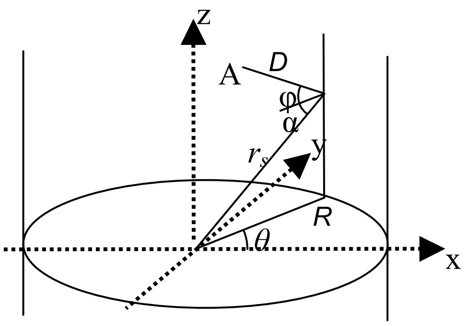


Figure 7. co-ordinates used for the cylinder

We will define: $r_s = \sqrt{R^2+z^2}$ and R= radius of the cylinder. For point A the distance to the radiating element *dS* will be *D*. The Kirchhoff integral will then be:

$$P_A = \frac{\hat{p}}{4\pi}\int_S\left(\frac{1+jkr_s}{r_s}\cos\alpha + \frac{1+jkD}{D}\cos\varphi\right)\frac{e^{-jk(D+r_s)}}{Dr_s}dS \quad (8)$$

or assuming source and receiver in the far field $(r_s >> \lambda, D >> \lambda)$:

$$P_A = -\frac{j\hat{p}}{\lambda}\int_S\left(\frac{\cos\alpha+\cos\varphi}{2}\right)\frac{e^{-jk(D+r_s)}}{Dr_s}dS \qquad (9)$$

For A=(0,0,0) this can be simplified, by using $D = r_s$ and $\cos\alpha = \cos\varphi = \frac{R}{D}$:

$$P_A = -\frac{j\hat{p}R^2}{\lambda}\int_0^{2\pi}\int_{-\infty}^{\infty}\frac{e^{-jk2D}}{D^3}dzd\theta = -j\hat{p}kR^2\int_{-\infty}^{\infty}\frac{e^{-2jk\sqrt{R^2+z^2}}}{(R^2+z^2)^{3/2}}dz$$

This integral is solved in [6] (for $R > \lambda$):

$$P_A = -j\hat{p}kR^2\int_{-\infty}^{\infty}\frac{e^{-2jk\sqrt{R^2+z^2}}}{(R^2+z^2)^{3/2}}dz = \hat{p}(1+i)\sqrt{\frac{\pi k}{2R}}e^{-2jkR}$$

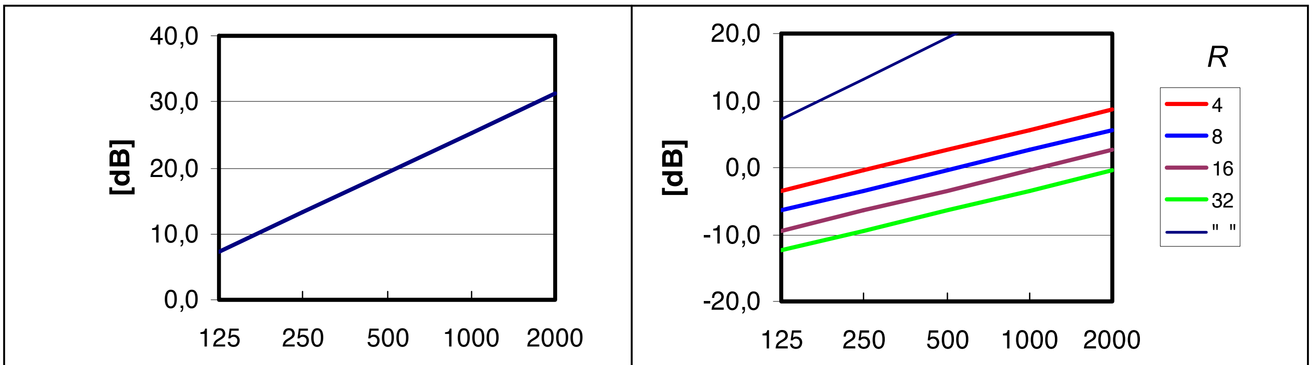
(10)

for half a cylinder this will result in:

$$|P_A| = \tfrac{1}{2}\hat{p}\sqrt{\frac{\pi k}{R}} \text{ or } p_{eff}^2 = \tfrac{1}{2}\hat{p}^2\frac{\pi k}{4R} \qquad (11)$$

Figure 8 shows the increase in sound pressure level of a hemisphere and a half-cylinder. It shows that especially for the hemisphere very high SPL's are possible, especially at high frequencies. In practical situations however the reflection is better audible at lower frequencies, firstly because the diffusion due to surface irregularities is more at high frequencies and secondly because the focal area is larger at low frequencies. Since even very good diffusers still have some energy specularly reflected [7], it will be difficult to suppress the reflection more than 10 dB [8]. This answers the second question put in the introduction, since it means that even with very good diffusion the increase of the SPL in the focal point will still be significant.

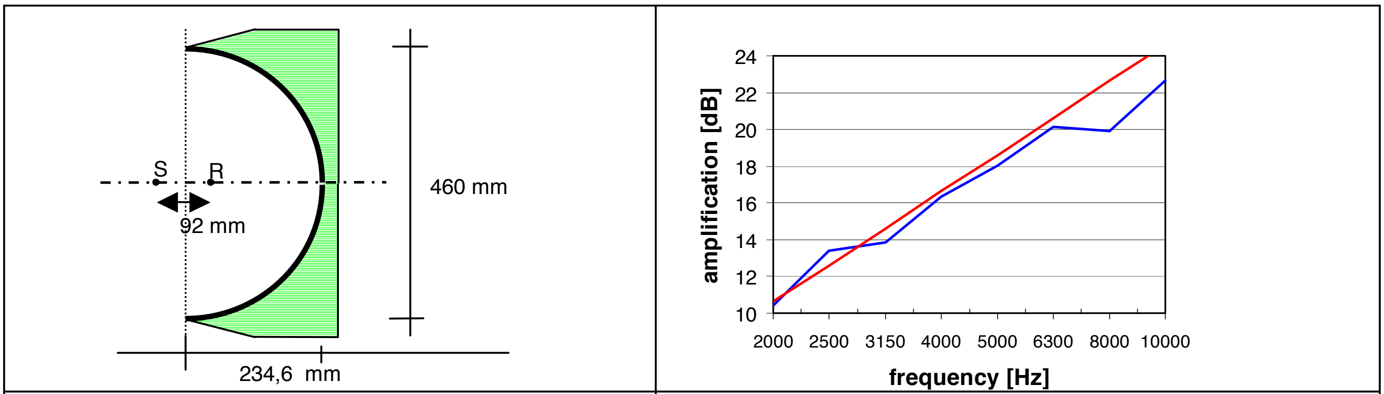
In figure 8 we also see that the increase of sound pressure level in a cylinder shape is much less than in a dome shape. Although the SPL can be significant, it seems feasible to reduce the reflection in the focal point to a reasonable level by diffusion (a further reduction of the level by 10 dB max.).



Hemisphere: $\Delta L = 10\log(k^2)$

Half Cylinder: $\Delta L = 10\log\left(\frac{\pi k}{4R}\right)$

Figure 8. Increase in sound pressure level ΔL in the focal point, relative to the SPL at 1 m distance from the source, for a hemisphere and a half cylinder for different radii (in m).



Section of the experimental setup (half ellipsoid), S=source, R=microphone

Difference between SPL reflected and direct sound, red: theory (7) and blue: measured

Figure 9. Experimental setup (left) and measurement result (right)

5 Experimental verification of the amplification in the focal point

To verify the theoretical amplification in the focal point (6) an experimental setup at small scale was made. It consists of an half ellipsoid with the two focal points at relatively small distance. The model is CAD/CAM milled from a solid polyurethane block (Ebaboard PW 920), a material with a high density and excellent low surface porosity. The accuracy of the shape of the ellipsoid is approx. 0.01 mm.

The impulse response was measured with a MLS (Maximum Length Sequence) signal. In time domain the separation of direct signal and (single) reflected signal was made. The setup and frequency dependant difference between reflected and direct sound pressure level are shown in figure 9. The results show a very good agreement with the theory.

6 Reflected sound field from a curved surface

Now we will concentrate on a rectangular surface that is curved in two directions (curvature defined by the radii R_y and R_z), see figure 10:

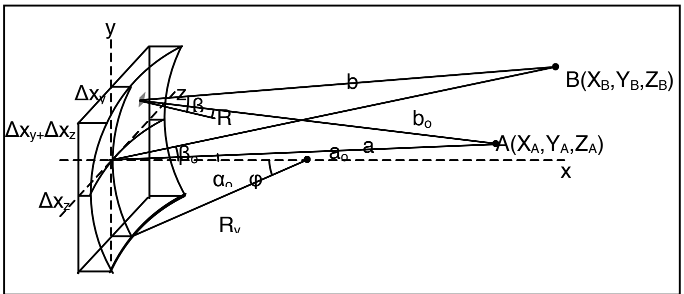


Figure 10: Co-ordinates and symbols used for the calculation of the reflected sound field form curved

In case the source A and the receiver point B are situated sufficiently far from the reflection object $(a_o, b_o > 2S/\lambda)$, then the amplitude function can be assumed independent from dS:

$$a \approx a_0\,, \cos\alpha \approx \cos\alpha_0\,,\; b \approx b_0\,,\; \cos\beta \approx \cos\beta_0$$

The Kirchhoff integral can be written as:

$$p_B = \frac{\hat{p}}{4\pi \cdot a_0 b_0}\left(\frac{1+jka_0}{a_0}\cos\alpha_0 + \frac{1+jkb_0}{b_0}\cos\beta_0\right)\iint e^{-jk(a+b)}dydz$$

For the phase function a more accurate description of the distance to dS is necessary. From the Taylor series the curvature Δx can be estimated:

$$\Delta x \approx R - R\cos\frac{y}{R} \approx R - R\left(1-\frac{y^2}{2R^2}\right) = \frac{y^2}{2R} \qquad (13)$$

This will result in:

$$a^2 = \left(X_A - \left(\frac{y^2}{2R_y} + \frac{z^2}{2R_z}\right)\right)^2 + (Y_A - y)^2 + (Z_A - z)^2 \qquad (14)$$

When we will neglect the quadratic terms of y and z, when we expand the root by a taylor series and when we will use:

a_o: $a_0^2 = X_A^2 + Y_A^2 + Z_A^2$, then we can approximate a by:

$$a \approx a_0 - \frac{X_A}{a_0}\left(\frac{y^2}{2R_y} + \frac{z^2}{2R_z}\right) - \frac{Y_A}{a_0}y - \frac{Z_A}{a_0}z \qquad (15)$$

This solution resembles the solution of the Fraunhofer diffraction [9], with the exemption of the quadratic terms due to the curvature. From (15) the phase function can be obtained:

$$-jk(a+b) = -jk(a_0 + b_0) + jC_1y^2 + jC_2y + jC_3z^2 + jC_4z$$

$$C_1 = k\left(\frac{X_A}{2a_0R_y} + \frac{X_B}{2b_0R_y}\right), C_2 = k\left(\frac{Y_A}{a_0} + \frac{Y_B}{b_0}\right),$$

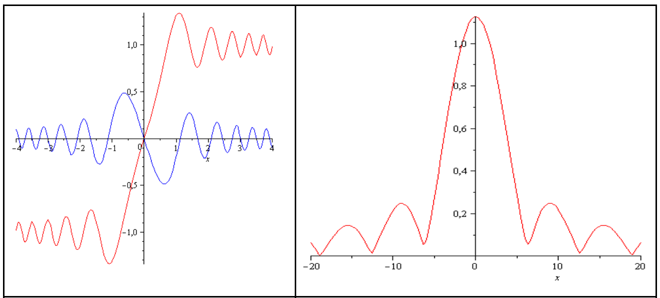
$$C_3 = k\left(\frac{X_A}{2a_0R_z} + \frac{X_B}{2b_0R_z}\right),\; C_4 = k\left(\frac{Z_A}{a_0} + \frac{Z_B}{b_0}\right) \qquad (16)$$

This phase function can be split up into y- and z-direction. The resulting integral can be solved [10]:

$$p_B = \frac{\hat{p}\cdot e^{-jk(a_0+b_0)}}{4\pi\cdot a_0 b_0}\left(\frac{1+jka_0}{a_0}\cos\alpha_0 + \frac{1+jkb_0}{b_0}\cos\beta_0\right)\cdot$$

$$\sqrt{\frac{\pi}{8C_1}}(1+j)\cdot e^{-j\frac{C_2^2}{4C_1}}\cdot\left[\operatorname{erf}\left(\frac{(1+j)}{\sqrt{8C_1}}(2C_1y_2-C_2)\right)-\operatorname{erf}\left(\frac{(1+j)}{\sqrt{8C_1}}(2C_1y_1-C_2)\right)\right] \quad (17)$$

$$\cdot\sqrt{\frac{\pi}{8C_1}}(1+j)\cdot e^{-j\frac{C_4^2}{4C_3}}\cdot\left[\operatorname{erf}\left(\frac{(1+j)}{\sqrt{8C_3}}(2C_3z_2-C_4)\right)-\operatorname{erf}\left(\frac{(1+j)}{\sqrt{8C_3}}(2C_3z_1-C_4)\right)\right]$$

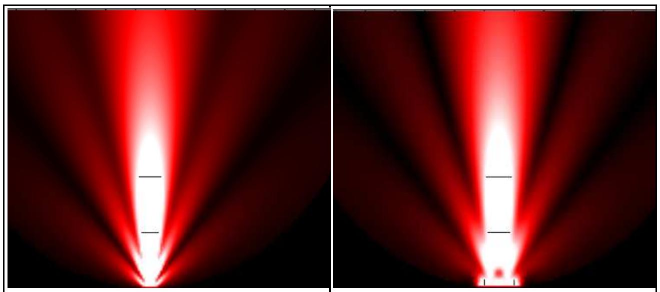


The errorfunction: $z = erf((1+j)\cdot x)$, Real (red) and Imaginary part (blue). x is a real number.	The difference of two error functions (amplitude): $z = erf\left(\frac{1+j}{\sqrt{8}}(1-x)\right) - erf\left(\frac{1+j}{\sqrt{8}}(-1-x)\right)$

Figure 11. The complex error function (left) and the difference between to shifted error functions (right)

The difference between two complex error functions is a function with a main lobe and side lobes, somewhat related to Bessel functions, see figure 11.

Figure 12 shows the calculation result of this analytical solution (17) and a numerical solution based on (1)



Calculation with (17)	Numerical solution with (1)

Fig.12 Reflected sound field by a curved surface of 1x1 m, radius 10 m, source in centre point, k=9,2 (500 Hz).

With this analytical formulation both pressure and phase of the reflected sound field can be calculated with sufficient accuracy.

7 Conclusions

From the presented data we can conclude that:

The (maximum) SPL in the focal point can easily predicted; for the sphere it is not dependent on the radius, for the cylinder it is. The focussing effect of the cylinder is much less then the sphere.

Since all diffusing objects still reflect part of their energy specularly, the means of reducing focussing effects by diffusing objects are limited. Strong echo's from hemispheres are likely to remain to some extent, even with good diffusion. Echo's from cylinder shapes are easier to remove.

The sound field at some distance from the source can be calculated by an extension of the Fraunhofer diffraction solution for curved surfaces.

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