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REFLECTIONS OF SOUND FROM CONCAVE SURFACES

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Vercammen, Martijn
Peutz bv, P.O. Box 66, NL-6585 ZH MOOK, Netherlands; m.vercammen@mook.peutz.nl

INTRODUCTION

Many small or large rooms have concave surfaces. With improved building technology and fashions in architecture (blobs) problems due to these surfaces are encountered more and more. Some situations are described in literature [1,2,3]. In our consultancy work we had to deal with these situations e.g. in concert halls [4,5].
When sound is reflected from a concave surfaces the geometry of the surface will force the energy to concentrate. The sound pressure due to this focussing if mostly calculated by computer simulation techniques applied on a segmented shape or by a geometrical approximation. Both methods however fail in the focussing point, the result is not even close to the real value.
To correctly estimate the sound pressure a wave extrapolation method is used, that will be presented here.

WAVE EXTRAPOLATION

Wave extrapolation uses the Huygens principle, developed by Christiaan Huygens in 1678 and later improved by Fresnal.The Huygens Principle states that every point on the primary wavefront can be thought of as an emitter of secondary wavelets. The secondary wavelets combine to produce a new wavefront in the direction of propagation. Fresnel extended the theory of Huygens in stating that the secondary wavelets mutually interfere. But it was Kirchhoff who put the Huygens-Fresnel principle on a sounder mathematical basis.
From Green's theorem the Kirchhoff integral can be derived:

$$P_A = \frac{1}{4\pi}\int_S \left(P(r)\frac{1+jkD}{D}\cos\varphi\frac{e^{-jkD}}{D} + j\omega\rho\cdot v_n(r)\frac{e^{-jkD}}{D}\right)dS =$$

$$\frac{\hat{p}}{4\pi}\int_S \left(\frac{1+jkD}{D}\cos\varphi + \frac{1+jkr}{r}\cos\alpha\right)\frac{e^{-jk(D+r)}}{Dr}dS \qquad (1)$$

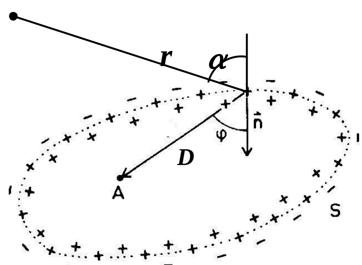

Figure 1. Symbols used for the Kirchoff integral

It states that, with a sound source outside volume V, the sound pressure at point A inside V can be calculated from the sound pressure and particle velocity data on the surface S that is the boundary of V.
In the integral above the sound pressure and particle velocity is assumed to be known and resulting from a source at distance r from the surface element dS.
The distance between dS and A is given by D. The angles α and φ are the angles between the incident sound and the (outer)normal to dS and between the (inner)normal to dS and the vector to A from dS respectively.
For $r >> \lambda$ and $d >> \lambda$ (far field) this can be simplified to:

$$P_A = -\frac{j\hat{p}}{\lambda}\int_S \left(\frac{\cos\alpha + \cos\varphi}{2}\right)\frac{e^{-jk(D+r)}}{Dr}dS \qquad (2)$$

This is known as the Fresnel-Kirchhoff diffraction formula.
In case of wave propagation from a flat surface the Kirchhoff integral may be further simplified to the Rayleigh integral, either using the sound pressure data or the particle velocity data. Since we will calculate the reflection by extrapolating the wavelets on concave surfaces we will use the Kirchhoff integral. This is done by first calculating the wavefield from the source on the concave surface and then (without the source

and with the velocity at opposite phase) integrating the contributions of the wavelets over the concave surface, thus obtaining (only) the reflected sound.

SPECULAR REFLECTION FROM A SPHERE

We take a full sphere with radius R and describe it with the spherical coordinates (r, ϕ, θ) and the surface elements

$dS = R^2 \cdot \sin\phi \cdot d\phi \cdot d\theta$.

If the sound source is in the origin, the sound pressure at the sphere's surface will be:

$$P(R,\omega) = \hat{p}\frac{e^{-jkR}}{R} \qquad (3)$$

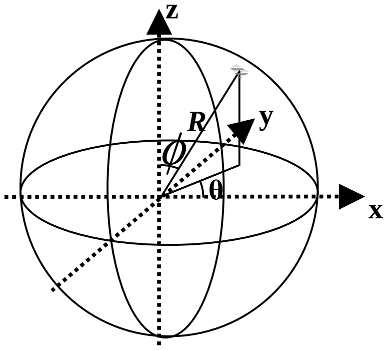

Figure 2. co-ordinates used.

Integrating over the sphere will give the following (Kirchhoff) integral expression for the pressure in a point A inside the sphere:

$$P_A = \frac{\hat{p}R \cdot e^{-jkR}}{4\pi}\int_0^{\pi}\int_0^{2\pi}\left(\frac{1+jkR}{R}\cos\alpha + \frac{1+jkD}{D}\cos\varphi\right)\frac{e^{-jkD}}{D}\sin\phi \cdot d\phi d\theta \qquad (4)$$

For the source in the origin $\cos\alpha = 1$. For point $A = (0,0,0)$ an analytical solution is possible (D=R, $\cos\varphi = 1$):

$$P_A = -\frac{j\hat{p}R}{\lambda} \cdot e^{-jkR}\int_0^{\pi}\int_0^{2\pi}\sin\phi \cdot \frac{e^{-jkR}}{R} \cdot d\phi d\theta = -\frac{j\hat{p}}{\lambda} \cdot e^{-j2kR}\int_0^{\pi}\int_0^{2\pi}\sin\phi \cdot d\phi d\theta = -j2\hat{p}k \cdot e^{-j2kR}$$

The amplitude of the reflecting wave in the center of the sphere is: $|P_A| = 2k\hat{p} = \frac{4\pi\hat{p}}{\lambda}$ (5)

This amplitude corresponds roughly with the amplitude one would get if the reflected energy is distributed over an circular area with radius $\frac{1}{4}\lambda$.

Since all contributions of dS are in phase, this amplitude is proportional to the reflecting surface that contributes. For a <u>hemi</u>sphere the amplitude in the center will be:

$|P_A| = k\hat{p}$, or $p_{eff}^2 = \frac{1}{2}\hat{p}^2 k^2$ (6)

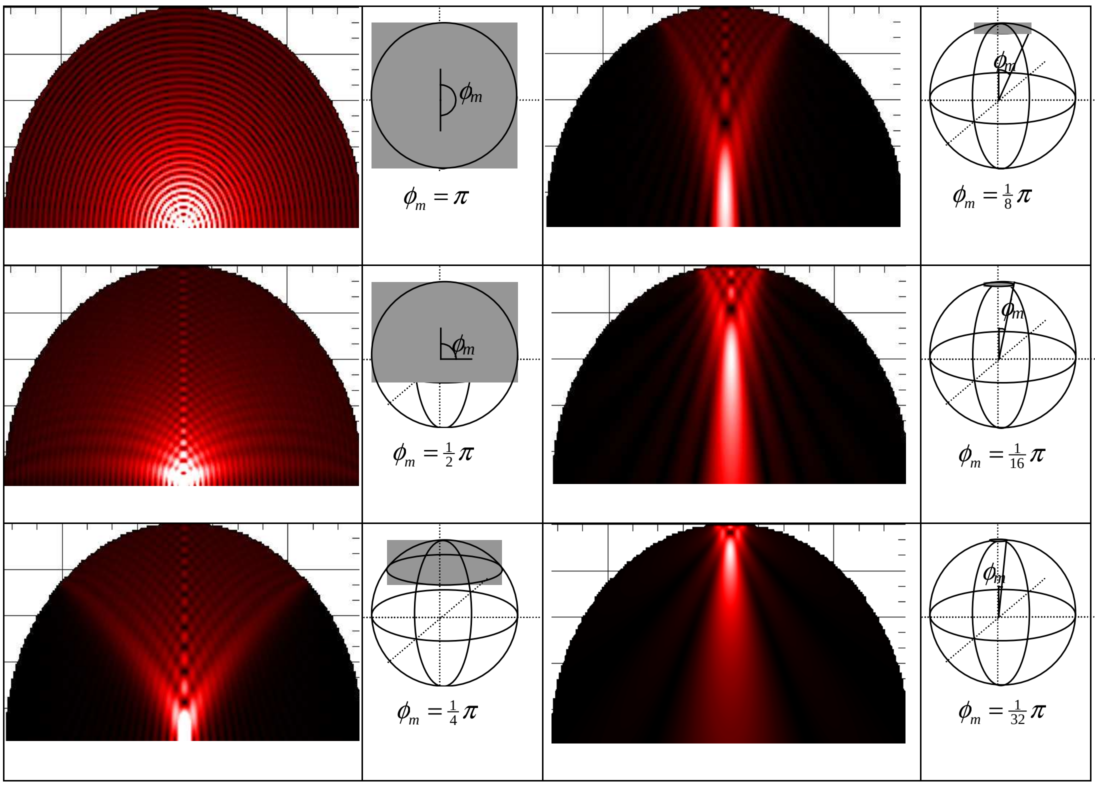

Figure 3. reflected sound pressure from a dome segment; R=10m, k=10 (f=500Hz); black=0, white ≥ 1.

For a sphere fragment integrating ϕ from *0* to ϕ_m this will result in

$$|P_A| = k\hat{p}(1-\cos\phi_m) = \frac{2\pi\hat{p}(1-\cos\phi_m)}{\lambda} \qquad (7)$$

In figure 3 some examples of the calculated pressure is given for different ϕ_m .

Clear interference patterns are visible. It is also clear that the energy will not concentrate in a single point. For smaller disk elements ($\phi_m \le \frac{1}{8}\pi$) a "beam" appears that will widen due to diffraction, depending on frequency and size of the dome segment. The width of the reflected "beam" (between the two first pressure dips) at the focal point can approximated by $b = \lambda / \sin(\phi_m)$ for $\phi_m \le \frac{1}{2}\pi$.

The "beams" can be approximated with Fraunhofer diffraction from a circular disk. Using for radius $a = R\sin\varphi_m$, the pressure at distance *r*, with offset *x* from the axe, can be calculated from:

$$P(x) = \frac{\hat{p}}{\lambda}\frac{\pi a^2}{r^2}\frac{2J_1(\frac{kax}{r})}{(\frac{kax}{r})} \approx \frac{\hat{p}}{\lambda}\frac{\pi a^2}{r^2}\cos(\frac{kax}{2r}) \qquad (8)$$

If one disregards the sidelobes, the main lobe of the Bessel function can be approximated by the given cosine (for $-\pi < \frac{kax}{r} < \pi$).

Figure 4 shows the calculated wave field from a small sphere segment, by extrapolation and by formula (8). Despite the differences in source position (for the sphere in the center and for Fraunhofer diffraction at infinity) the sound field is very similar.

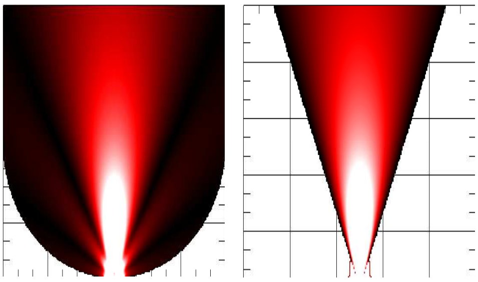

Figure 4. Simulation of the reflected sound field (f=500 Hz, $\lambda = 0{,}68$ m) from:
left: a sphere segment $\varphi_m = \frac{1}{32}\pi$,R=10 m (4)
right: a disk a=1m (8)

SPECULAR REFLECTION FROM A CYLINDER

The reflected sound field from a cylinder can, as for the dome, be described by the Kirchhoff integral.
The positions within the cylinder are described by cylindrical co-ordinates (r,θ,z) and the surface elements on the cylinder have dimension $dS = Rd\theta dz$. The source is assumed in the center (0,0,0).

We will define: $r_s = \sqrt{R^2+z^2}$ and R= radius of the cylinder.

For point A the distance to the radiating element dS will be *D*. The Kirchhoff integral will then be:

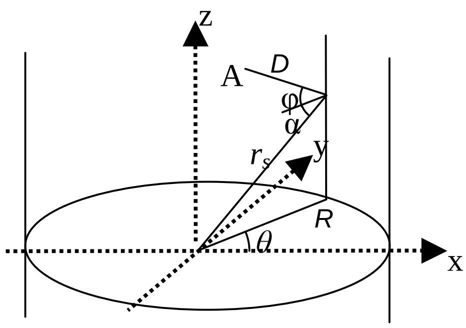

Figure 5. co-ordinates used coordinates

$$P_A = \frac{\hat{p}}{4\pi}\int_S \left(\frac{1+jkr_s}{r_s}\cos\alpha + \frac{1+jkD}{D}\cos\varphi\right)\frac{e^{-jk(D+r_s)}}{Dr_s}dS \qquad (9)$$

or assuming source and receiver in the far field $(r_s >> \lambda, D >> \lambda)$:

$$P_A = -\frac{j\hat{p}}{\lambda}\int_S \left(\frac{\cos\alpha+\cos\varphi}{2}\right)\frac{e^{-jk(D+r_s)}}{Dr_s}dS \qquad (10)$$

For A=(0,0,0) this can be simplified, by using $D = r_s$ and $\cos\alpha = \cos\varphi = \frac{R}{D}$:

$$P_A = -\frac{j\hat{p}R^2}{\lambda}\int_0^{2\pi}\int_{-\infty}^{\infty}\frac{e^{-jk2D}}{D^3}dzd\theta = -j\hat{p}kR^2\int_{-\infty}^{\infty}\frac{e^{-jk2D}}{D^3}dz = -j\hat{p}kR^2\int_{-\infty}^{\infty}\frac{e^{-2jk\sqrt{R^2+z^2}}}{(R^2+z^2)^{3/2}}dz$$

This integral is solved in [6] (for $R > \lambda$)

$$P_A = -j\hat{p}kR^2\int_{-\infty}^{\infty}\frac{e^{-2jk\sqrt{R^2+z^2}}}{(R^2+z^2)^{3/2}}dz = \hat{p}(1+i)\sqrt{\frac{\pi k}{2R}}e^{-2jkR} \qquad (11)$$

for half a cylinder this will result in:

$$|P_A| = \tfrac{1}{2}\,\hat{p}\sqrt{\frac{\pi k}{R}} \text{ or } p_{eff}^2 = \tfrac{1}{2}\,\hat{p}^2\,\frac{\pi k}{4R} \tag{12}$$

DIFFUSE REFLECTION FROM A SPHERE

Non-specular reflections may have specific direction, depending on the surface structure, but if this is not the case, the directional characteristics of a Lambert radiation may be assumed.

We assume that for diffuse reflections in each reception point there is random phase relation so we will add energy in stead of pressure.

The Intensity of Lambert radiation is dependent on the angle φ with the normal to the radiating surface: $I_\varphi = I_0 \cdot \cos\varphi$

We have a source in the center of a hemisphere, the incident intensity on the surface element dS of the hemisphere with radius R is:

$$I = \frac{\hat{p}^2}{2\rho c \cdot R^2} \tag{13}$$

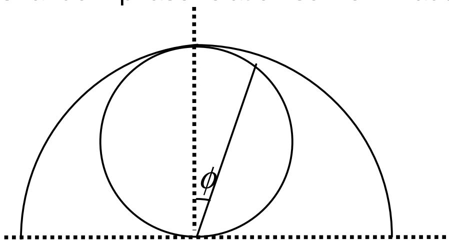

Figure 6. Lambert radiation

Assuming all incident energy is reflected, the intensity in a point at angle φ and distance D from surface element dS can be calculated from:

$$I = \frac{\hat{p}^2 dS\cos\varphi}{2\rho c R^2 \pi D^2}, \text{ or } p_{eff}^2 = \frac{\hat{p}^2 dS\cos\varphi}{2R^2\pi D^2}$$

The total pressure results from integration over dS:

$$p_{eff}^2 = \int_s \frac{\hat{p}^2}{2\pi}\frac{\cos\varphi}{R^2 D^2}\,dS \tag{14}$$

In the center of the hemisphere this reduces to $(R = D, \cos\varphi = 1)$:

$$p_{eff}^2 = \frac{\hat{p}^2}{2\pi R^4}\int_s dS = \frac{\hat{p}^2}{R^2} \tag{15}$$

DIFFUSE REFLECTION FROM A CYLINDER

Firstly we will assume that the surface element on the cylinder is diffusely reflecting all energy. We will also assume Lambert radiation characteristics.

The intensity in a point at angle φ and distance D from surface element dS can be calculated from:

$$I = \frac{I_i dS\cos\alpha\cos\varphi}{\pi r_s^2 D^2} \tag{16}$$

Where I_i = the intensity of the incident wave: $I_i = \dfrac{\hat{p}^2}{2\rho c \cdot r_s^2}$

The total pressure results from integration over dS:

$$p_{eff}^2 = \int_s \frac{\hat{p}^2}{2\pi}\frac{\cos\alpha\cos\varphi}{r_s^2 D^2}\,dS \tag{17}$$

If we consider the pressure in the origin this reduces to $(r_s = D = \sqrt{R^2 + z^2}, \cos\alpha = \cos\varphi = \dfrac{R}{D})$:

$$p_{eff}^2 = \frac{\hat{p}^2}{2\pi}R^3 \int_0^{2\pi}\int_{-\infty}^{\infty} \frac{1}{D^6}\,dz\,d\theta = \hat{p}^2 R^3 \int_{-\infty}^{\infty} \frac{1}{(R^2+z^2)^3}\,dz \tag{18}$$

it can be shown that the solution of this integral is: $p_{eff}^2 = \tfrac{3}{8}\pi\dfrac{\hat{p}^2}{R^2}$ for the full cylinder,

or for the half cylinder: $p_{eff}^2 = \tfrac{3}{16}\pi\dfrac{\hat{p}^2}{R^2}$ (19)

SCATTERING BY DIFFUSERS

In case of diffusers attached to the sphere or cylinder the question will be how much of the incident energy will be reflected specular and how much as Lambert radiation (see figure 7).

The ratio of scattered (non-specular) energy to total energy is described by the scattering coefficient. This is one minus the ratio of specular reflected energy to total reflected energy, so:

$$s = 1 - \frac{E_{spec}}{E_{total}} \qquad (20)$$

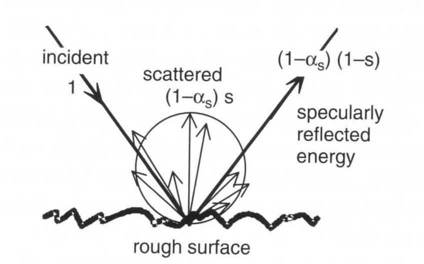

Figure 7. The non-absorbed energy is either specularly reflected or scattered [7].

The total energy in the center can thus be calculated from the specular and diffuse contributions, for the hemisphere that will be:

$$p_{eff}^2 = \tfrac{1}{2}\hat{p}^2\{(1-s)k^2 + \frac{s}{R^2}\} \qquad (21)$$

The scattering coefficient can be measured or calculated, although calculating is not an easy task. In [7] for a number of diffusers the scattering coefficient is calculated using BEM (Boundary Element Method). The calculated scattering of some relatively good diffusers is given in figure 8. For good diffusers the scattering coefficient can reach values of approx. 0,9 for the high frequencies. That means that it is difficult to suppress specular reflections more than 10 dB (s=0,9).

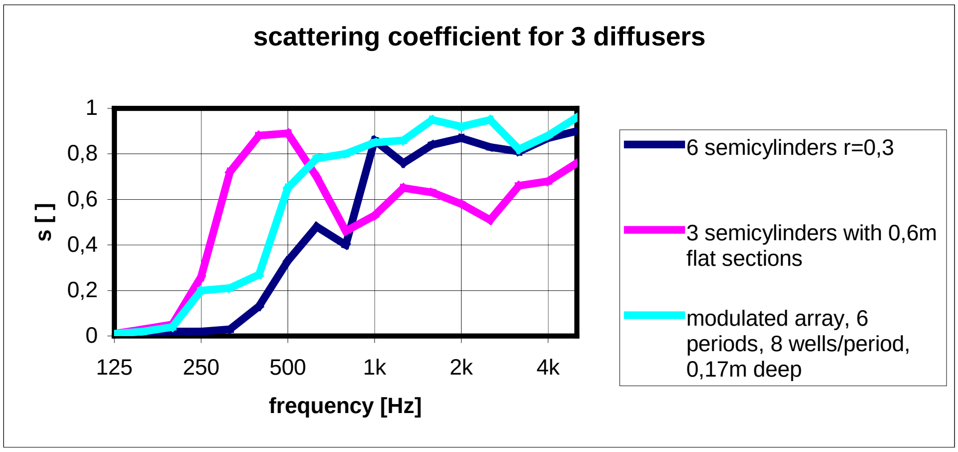

Figure 8, the calculated scattering using BEM, from [7]

OVERVIEW

The increase ΔL in sound pressure level in the focal point compared to the SPL at 1 m from the source, in a hemisphere and a half-cylinder, can be calculated using the formulas given in the table below.

	Hemisphere	Half cylinder
Specular reflections	$10\log\left(k^2\right)$	$10\log\left(\frac{\pi k}{4R}\right)$
Diffuse (Lambert) reflections	$10\log(\frac{1}{R^2})$	$10\log\frac{3}{16}\frac{\pi}{R^2}$
Specular + diffuse reflections	$10\log\{(1-s)k^2 + \frac{s}{R^2}\}$	$10\log\{(1-s)\left(\frac{\pi k}{4R}\right) + \frac{3}{16}\frac{\pi \cdot s}{R^2}\}$

The relation between ΔL depending on radius R and frequency is given in figure 9 for frequencies 250,500 and 1kHz. For combined specular and diffuse reflections scattering coefficients 0.2,0.65 and 0.85 are assumed.

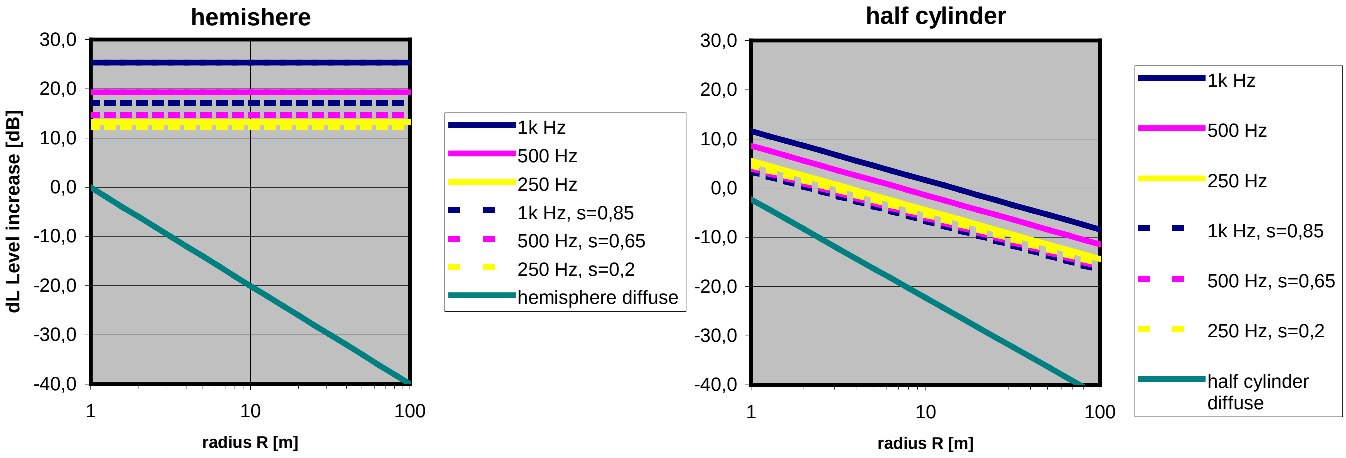

Figure 9. Sound pressure level increase ΔL (relative to the SPL 1 m) in the focal point of a hemisphere (left) and a half cylinder(right). The calculation results are given for specular reflections at 3 frequencies (solid lines), diffuse reflections (green lines) and combination of specular and diffuse reflections (dotted lines).

CONCLUSIONS

From the presented data we can conclude that:
The (maximum) SPL in the focal point can easily predicted; for the sphere it is not dependent on the radius, for the cylinder it is. The focussing effect of the cylinder is much less then the sphere.
The contribution of diffuse reflections to the SPL in the focus point is small compared to the influence of specular reflections.
Since all diffusing objects still reflect part of their energy specular, the means of reducing focussing effects by diffusing objects are limited. Strong echo's from hemispheres are likely to remain to some extent, even with good diffusion. Echo's from cylinder shapes are easier to remove.

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