Influence of sample size on the sound absorption

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Introduction

The sound absorption of materials not only depends on material properties but also on the geometrical properties and the incident sound field. The absorption of materials is usually determined in a reverberation chamber on a sample of certain dimensions, within a limited range. The measurement is made according to ISO 354. The finite size of the absorber may result in absorption coefficient well over 1 and in practical situations, for different dimensions, the actual absorption will be different. This size related problem is addressed in this paper. Purpose is to estimate the absorption of an infinite sample and the absorption of a finite sample with different dimensions, from the data of the reverberation room measurement.

Experimental data

Between 1960 and 1980 there has been several investigations into the effects of geometry on the absorption, e.g. [2]-[5]. Usually it is referred to as the edge effect or area effect. The outcome of this research was that if one varies the size of the sample there seems to be a linear relation between the relative “edge length” E=L/S (L is total length of the edges, S is surface of the sample) and the measured sound absorption (see figure 1). Extrapolation to E=0 leads to the sound absorption of an infinite sample \( \alpha_\infty \) for random incidence:

\[
\alpha_s = \alpha_\infty + \beta E
\] (1)

Where \( \alpha_s \) = measured sound absorption in the rev. room.

The derived sound absorption for an infinite sample seemed to correspond fairly well with the calculated sound absorption for random incidence based on measurement of the impedance in the interferometer tube and Paris’ formula [3]. The idea was that from measurements with different sample size the \( \beta \) could be determined and from that the \( \alpha_\infty \) as relevant parameter could be determined. This proposal didn’t make it into the standard ISO 354. The standard also did not fix the \( E \) either, although the requirements on the sample (S between 10 and 12 m\(^2\), width to length ration between 0,7 and 1) results in an \( E \) within 1,13 and 1,29 m\(^3\). It has to be noted that the \( \beta \) is dependant on the material, especially on the sound absorption coefficient. For low sound absorption the \( \beta \) is also low. Figure 2 shows the \( \beta \) found in [3]. An accurate experimental determination of \( \beta \) is not easy. For that at least two measurements with significantly different \( E \) are needed. The measurement for high \( E \), with large edge length and small surface area requires more than one sample, e.g. strips, and these samples might influence each other. Furthermore the use of concept using the edge length as a parameter may be questionable, since for strips the edges can hardly be considered as independent from each other, especially not for low frequencies. So a valid theoretical determination of the influence of samples size on sound absorption will be very helpful.

Theoretical model for the absorption

The angle dependant absorption of infinite, locally reacting materials, with impedance \( Z_A \) is described by (see [1],[9]):

\[
\alpha_\omega (\theta) = \frac{4 \text{Re}(Z_A) \cos(\theta)}{|Z_A + 1/\cos(\theta)|} \]

(2)

There is a singularity in this expression for grazing incidence. The absorption for random incidence can be determined by integration over the half sphere, leaving out the grazing incidence angles. In [8] and [9] this theory is extended for finite materials. Based on a variational approach the angle dependant absorption for highly absorptive materials can be approximated by:

\[
\alpha_\omega (\theta) \approx \frac{4 \text{Re}(Z_A) \cos(\theta)}{|Z_A + Z_F|} \]

(3)

The field impedance \( Z_F \) is actually the radiation impedance of the sample for the situation that the velocity on the sample corresponds to the incident field due to a plane wave with angle \( \theta \) and can be calculated from [9] (see also [10]):

\[
Z_F = \frac{jk}{2\pi} \int_{s} \int_{s} \int_{s} -j\beta (im/\omega)(e^{-j\omega t}+e^{j\omega t})\delta(x-x_0)\delta(y-y_0) dx_0 dy_0 dx \]

(4)

where \( A \) = sample area and \( k \) is the wave number \( \omega c \).

In [9] it is shown that the absorption for random incidence for an infinite sample relates to the absorption of a (highly absorbing) finite sample:

\[
\alpha_\omega = \alpha_\infty / K \]

(5)

with:

\[
K = \frac{1}{\pi} \int_{0}^{\pi/2} \sin \theta \text{Re}(Z_F) d\theta \]

(6)
Interesting property of $K$ is that it only depends on the geometry of the sample and frequency. That means that $K$ can be determined for different geometry’s beforehand and can be used to calculate the absorption of an infinite sample. Table III in [9] contains results of numerical calculations of $K$ as a function of $ke$ for squares, where $e$ is the edge length. For rectangle shapes the average edge length can be determined from $e=2ab(a+b)$, with $a$ and $b$ the width and length of the rectangle (note that $E=4/e$). For values $1.4 < ke < 128$ the value of $K$ for square surfaces can be approximated by a polynomial regression ($R = 0.993$), that enables easy interpolation:

$$K = \frac{4.5}{(ke)} + \frac{6}{ke} + 1$$

(7)

Another approach could be to virtually extend the surface along the edges with a strip with width $\lambda/4$. The additional surface would be $ke$, with $\lambda$ for the wavelength. The ratio $K'$ of total (virtual) surface to actual surface will be:

$$K' = 1 + \frac{2\pi}{ke}$$

(8)

This expression is rather close to the numerical data in [9] for $ke > 2$ ($R = 0.997$), see figure 3.

![Figure 3](image)

**Figure 3**: Value of $K$ according to [9] (table III), the polynomial approach with (7) and the geometrical approach with (8).

### Estimation method

To determine the (theoretical) absorption of an infinite sample, (5) can be applied, using the measured absorption $a_s$ in the reverberation room. This relation is however based on a highly absorbing locally reacting homogeneous sample. It is also possible to calculate the absorption for an absorbing element of different size as the sample in the lab. In [9] a linear dependency of the absorption coefficient on the effect for finite sample of different size is assumed, resulting in:

$$a_i = a_s \frac{K}{K_L} + (1 - a_s)$$

(9)

where subscript $L$ refers to the laboratory situation.

### Comparison with experimental data

Some experimental data from [3] and [5] is further analysed. From these publications the absorption data, size of the sample and $\beta$ is known. Additionally measurements have been performed on a 10 cm mineral wool sample in 4
different set-ups, for $1.2 < E < 3.3$, for which the $\beta$ is determined. Based on the measured $a_s$ and the standard size of the sample, the absorption of an element with half the edge length is calculated with (9). The $\beta$ is then calculated according to (1) from $\beta = (\alpha_e - \alpha_{inf})(E-E_L)$. The calculated values can be compared to the measured ones in [3], [5] and the laboratory, see figure 4.

The results show that the tendency seems to be present, but the data do not match exactly. This does not mean that the calculations are not correct. Especially the fact that the samples may influence each other and opposite edges at short distance can hardly be considered independent, limits the validity of the experimental results.

![Figure 4](image)

**Figure 4**: Measured (markers) and calculated $\beta$ for: Left: 5,2 cm of glass wool [5], Middle: Silan [3], right: 10 cm mineral wool (Peutz acoustic laboratory)

### Conclusions

Mainly based on reference [9] an estimation method is presented to calculate the absorption of an infinite sample and a correction method for samples of different size.

### References


